The Mind Represents Space in Multiple Formats Simultaneously

by Diego Miró Rivera

Advisor: Frank Keil

Mentor: Sami Yousif

A Thesis

Submitted to the Department of Cognitive Science Yale University

In Partial Fulfillment of the Requirements

For the Degree of Bachelor of Science

April 28, 2023

ABSTRACT:

The qualities in the spaces we inhabit seem to impact the mental representations used to solve visuospatial problems. This study builds on recent work by Yousif & Keil (2021) which reveals the spontaneous use of a polar format system through error observation. We designed a location-placement task in which participants are asked to recall a location in space relative to either a landmark, boundary, or both. Each condition is designed to encourage the use of distinct mental formats. Our study goes one step further to consider if the mind can represent multiple formats simultaneously. Our results, which observe both accuracy and error correlation, show that participants can encode two formats at once, but will typically utilize the format, or combination of formats, most favored by the present condition. The analysis also reveals how a format influences errors in the participants' responses. For the landmark condition, results showed gravitation towards the landmark. For the boundary condition, results showed a significant gravitation to the nearest two points on the boundary, and the strongest gravitational pull towards the nearest boundary corners. These types of gravitational effects were not observed in the ‘both’ conditions. Ultimately, this study becomes another example of how spatial tasks are effective for observing the elusive, adaptative, and dynamic nature of representations in the mind.

INTRODUCTION:

The Ubiquity of Spatial Representations:

One of the most influential ideas in cognitive science in the last century is the idea of a *cognitive map*. A term coined by Edward Tolman (1948), a ‘cognitive map’ refers to a visuospatial map inside the mind, which corresponds to the external spatial world. Cognitive maps may refer to actual maps of the spatial environment, or to ‘mental maps’ of other feature spaces (e.g., number, as in Dehaene et al., 1993; social relations, as in Parkinson et al., 2013; and conceptual spaces as Peer et al., 2021). But what is the nature of the cognitive map? Does the cognitive map refer to one thing or many? How many distinct forms of ‘maps’ might there be?

There have been various ideas about how cognitive maps are structured. One common distinction is between *metric, Euclidean* geometric modules, (i.e., maps that preserve the geometry of the environment) (Kelly, 2018; Gallistel, 1990) and non-metric, graph-like cognitive maps (i.e., maps that preserve some geometric properties, like angle information, at the expense of some others; Kuipers, 1978; Kuipers, 1982; Warren et al., 2017). In addition to this basic distinction, there have been many other proposals that lie somewhere in between (e.g., Warren et al., 2017; see also Peer et al., 2021; Yousif, 2022). What these perspectives have in common is an assumption that the cognitive map has one underlying form – that the cognitive map *either* operates in one format or another. Is this necessarily true or can cognitive maps take various forms, perhaps even at the same time in one mind?

The idea that multiple representations can be held at once is reminiscent of the 'cognitive collage' proposed by Barbara Tversky (1993). As opposed to previously rigid and singular theories of mental representation, Tversky posed a compelling way of considering how the mind often holds an amalgamation of environmental knowledge at once. More recently, Sami Yousif (2022) has made an even stronger argument: that spatial information can be organized in multiple ways simultaneously. Yousif argues that there seems to be "redundant" formats which support the flexible spatial behavior observed in humans and that researchers should approach the study of all mental representations with this possibility in mind (Yousif, 2022). Until now, however, this has mostly been conjecture. There has not been a clear indication that multiple forms of spatial representation are stored redundantly and simultaneously. The goal of the present thesis is to address this possibility.

Finding Format:

Representations contain information, and that information must be organized, or *formatted*, in some way. Indeed, understanding how information is formatted in the mind is essential to understanding how the mind works in the first place. In cognitive science, format has been studied in a variety of ways. Stephen Kosslyn (2006) described format as a "type of code", much like different languages can be seen as a code that conveys content. The notion of format has shaped many high-profile debates. Perhaps most famously, Kosslyn and Pylyshyn argued about the nature of the format of mental imagery; with one describing it as an entity to be perceived (Kosslyn, 1996), and the other as a more conceptual and propositional understanding (Pylyshyn 1973). More contemporary work on the variance of mental imagery, with the extreme absence being aphantasia (the inability to visualize), may begin to explain the intensity and length of the Kosslyn and Pylyshyn debate over mental imagery.

For the sake of clarity on how mental formats might work, we could consider some concrete, spatial, formats that get used in our daily lives. For example, this paper has a format that follows the format of other scientific papers, communicating information on a white rectangle with an aspect ratio of 8.5 x 11, with a title at the top, with authors' names, and rows of smaller black text, with references at the end. This is a different format than other formats of communication, such as an Instagram post, an email, or a postcard. These external formats, which use visuospatial relationships graphics and other specific items for communicating information, are relatively easy to recognize and conceptualize.

Though the formats of mental representations are more obscure, they can often be revealed through surprisingly simple studies. For example, Firestone and Scholl (2016) showed that medial axis representations of shapes can be recovered from simple taps made on a screen. In their study, hundreds of unique observers simply tapped within a shape (e.g., a rectangle) a single time. Strikingly, the cumulative distribution of those taps revealed a highly systematic structure, which, for many unique shapes, strongly resembled the shapes' medial axis and provided a robust model of object representation (Firestone 2016; Ayzenberg et al. 2016).

The ability of the format to reveal itself through such simple tasks and errors is a common theme, not only in humans but in other animals as well. Mental representations aren't always accurate, and consistency in errors can be exploited to reveal patterns in how the representations are working. For example, a study by Müller et al., manipulated the length of desert ants' legs, to force an error that the researchers could observe. When putting stilts on the ants' legs they overshot their desired location; on the contrary, when their legs are cut short they undershoot their destination. This study revealed how the critical input of the ants' navigational representation was based on the relative distance of their stride to the terrain (Müller & Wehner, 1998).

To further understand how the mind is representing space, studies can go beyond observing behavior, and consider the shape that spatial information is taking in the brain. Research done on grid and place cells in the hippocampus made great progress in localizing and identifying how the brain is encoding spatial information; and perhaps revealing themselves as the smallest building blocks of spatial representations (Hartley, 2014). However, these invasive techniques, with direct contact to cells in the brain, cannot be replicated with humans.

Understanding the format of mental representations is essential for bridging the gap between brain and behavior. For instance, we know that at the highest levels of representation, the mind represents something akin to a cognitive map, and we know that at the lowest levels of representation, the mind represents space using grid cells, place cells, etc (O'Keefe et al., 1979). What we don't know, however, is how it is that these representations at the level of grid cells and place cells lead to forming full-fledged cognitive maps that are experienced, updated, and referenced in real-time. To understand this link, we must understand the format of all cognitive representations that lie in between neurons and cognitive maps (to understand a computer, we would have to understand all the languages, or formats, that exist between the file formats that we interact with, and the binary format of code implemented on the hardware of a device).

The Frame Problem: Boundaries vs. Landmarks

Everything in the universe is spatially relative to something else. The mind constantly must choose a frame, within a broader macrocosm, to represent how some things spatially relate to others. But, given multiple possible frames, what frame is chosen in each context or why?

We can begin to answer this by considering a classic distinction in spatial cognition; landmarks and boundaries. Narrowly defined, a boundary delineates a space, and a landmark represents a place. Various studies have explored if navigation is based on landmarks, boundaries, or both. For example, one study provided these two types of spatial cues, landmarks, and boundaries, and observed differences in navigation strategy revealed in children which revealed the use of multiple cues at once and changed their relative weighting as they grow older (Bullens, 2009).

Another study, using fMRI, found compelling results for the distinction between landmark and boundary, both from neural correlates and behavior. The right posterior hippocampal activation was found to reflect boundary-related locations, whereas right dorsal striatal activation reflected landmark-related locations. The behavioral aspect of the study showed landmark-learning used associative reinforcement, while boundary-learning was incidental, describing their role in a cognitive map (Doeller and Burgess 2008).

Ultimately the distinction between landmarks or boundaries can be exploited for various kinds of visuospatial experiments. Studies thus far have indicated that there is some distinction in how the brain encodes landmarks and boundaries. If people do separately represent spatial locations relative to landmarks and boundaries, then might we also expect that these separate representations have distinct formats?

Current Study:

This study is designed expecting landmarks and boundaries to have different formats. We propose that representations relative to landmarks are more likely to operate in a graph-like or polar format and expect that representations relative to boundaries are more likely to operate in a Cartesian format. The goal is to show this by observing accuracy between conditions, and error correlations.

The broader question our study is aimed at is not only revealing this distinction but also observing if these distinct formats can be represented in the mind simultaneously. This study is a location placement task. In summary, participants are shown a location, the point disappears, and they are then required to recall the location of that point relative to varying graphical elements. In this study, we will be observing the effects of three types of compositions, one that contains only a landmark, another with only a boundary, and one with both. We will observe the role that landmarks, boundaries, or both have on accuracy and analyze the errors made to consider how both formats behave.

This study is modeled after a recent work by Yousif & Keil, which has shown that the default 'format' of visuospatial representation may be polar coordinates (i.e., angle/distance relations) as well as unique methods to observe these formats (Yousif 2021). When provided minimal structure, errors reveal a default use of a polar coordinate system, however, when a highly structured environment is used, the coordinate system is deployed (Yousif 2021).

METHODS

Participants:

40 participants were recruited via Yale Psychology Subject Pool, this sample size was pre-registered. 3 additional participants were excluded (and replaced) for two reasons; if they didn't complete all 180 trials (2 exclusions) or if their overall accuracy was greater than 2.5 standard deviations above the mean (1 person). All participants were undergraduate students at Yale University, who completed the task on a desktop computer in our lab.

Display & Procedure: Encoding and Retrieval

The experiment was coded using custom software written in JavaScript. The functional part of the display consisted of a 1000 x 800 pixel region in the center of the screen. For encoding, participants are shown the stimuli; a gray dot (25 pixels in diameter) and a smaller blue dot (10 pixels in diameter) within a black square border (420 pixels). These graphical elements will appear in one of the four corners of the display (approximately 40 pixels away from the nearest horizontal and vertical borders of the display) at random (even distribution) during encoding. Each dot has a black border around it to maximize contrast and will always appear within the black border. The gray dot always appears in the bottom left corner of the square (approximately 40 pixels away from the nearest horizontal and vertical borders of the square), while the location of the blue dot varies. See Figure 1A.

The display is shown for 1s , then disappears, and .5s after, showing either the gray dot, the border, or both will reappear in the opposite corner of the display (with the same 40 pixel buffer) as the encoding phase. During this retrieval phase, the participants are asked to match the location of the blue dot relative to the other graphical elements, as they saw it in the previous display. They do this by clicking their mouse, with the option to click additional times or drag and drop the dot to change its location. There is no time limit to answering. Instead, whenever the participant is satisfied with their response, they are instructed to press the spacebar to continue to the next trial. Participants can view their progress in a progress bar on the top of the screen.

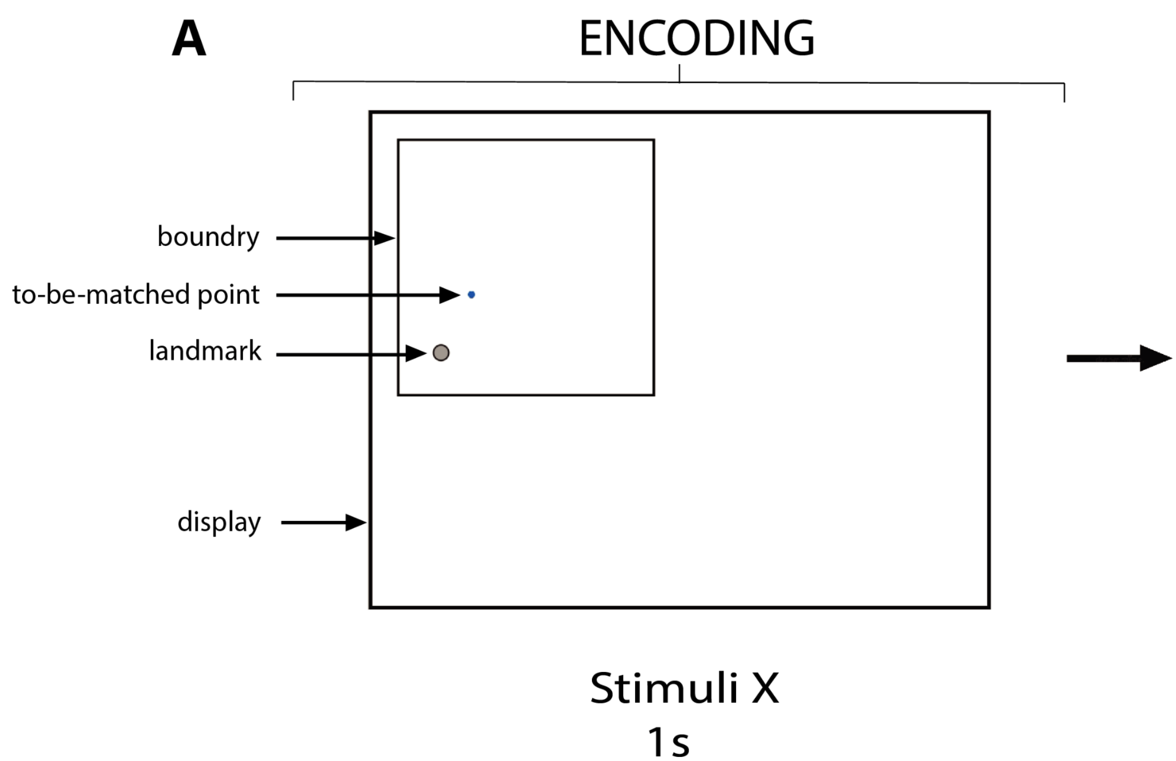

Figure 1A: shows an instance of one of the 180 trials. The stimuli consist of all the graphical elements whose locations are to be encoded (boundary, to-be-matched point, border within the display).

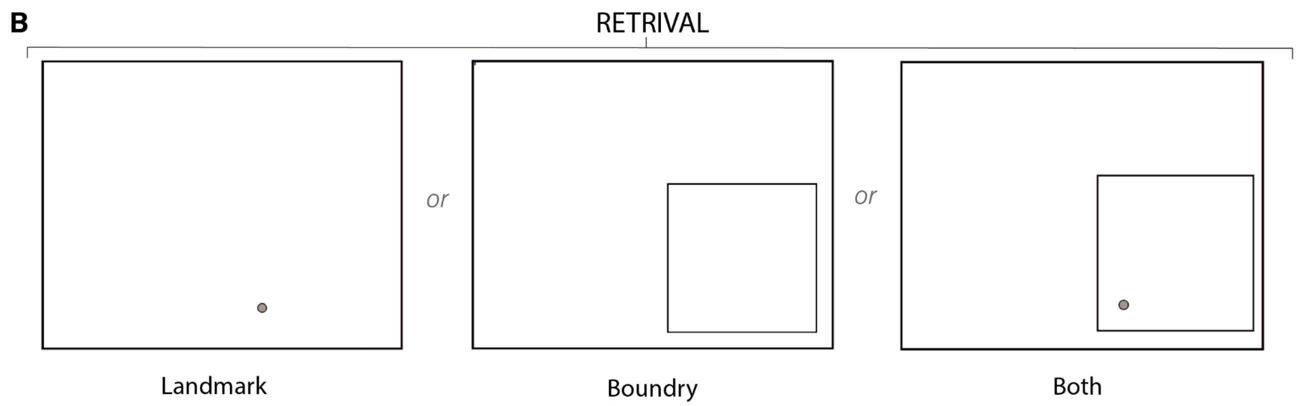

Figure 1B: shows the three trial types which could be shown after a given trial. Participants are asked to click where the blue dot should be, relative to the shown graphical elements. Note that the graphical elements in the retrieval appear in the opposite corner of the display shown during encoding.

Trials:

Participants completed 180 trials, which were divided into three trial types. Each trial type represents one configuration of the graphical elements at the time of retrieval. For one-third of the trials, only the landmark (gray dot) would be available at the time of retrieval; for another one-third, only the border (black square); and for the final one-third, both the landmark and the boundary would be available (see Figure 1B). The different trial types were intermixed in a fully randomized fashion (with no constraints). Participants completed two representative practice trials (the data from which were not recorded) before beginning the task.

Analysis:

We conduct three kinds of analyses. Using a combination of Excel and MatLab. The first is a primarily descriptive analysis (e.g., looking at average accuracy across the conditions, looking at compression towards or expansion from the landmark across conditions).Second, an 'error correlation' analysis. The core conclusions of this study rest on an analysis of the participants' errors (Yousif & Keil 2021). In short, we measure the correlation between errors in different dimensions of space (e.g., x vs y for cartesian coordinates, angle vs. distance for polar coordinates). If participants represent space via any 2D coordinate system, and the system is efficient, those two dimensions should be orthogonal. We propose, therefore, that errors in those two dimensions should also be orthogonal (for an example of this kind of analysis, see Bays et al., 2011). If participants represent space via Cartesian coordinates, we expect that their errors in this coordinate system would be independent, or uncorrelated. Similarly, if participants represent space via Cartesian coordinates, we expect that errors in other coordinate systems (e.g., polar coordinates) would be dependent, or correlated.

Third, we use a standardization and transposition procedure to visualize not only *how* people made their errors but *why.* We speculate which aspects of the visual composition would influence a participant's results. We analyze the gravitational effects of the landmark, the nearest points from the target to the boundary, and the nearest corner of the boundary to the target. All of these effects are considered relative to the effects seen in both conditions. More details on the calculations behind this analysis are articulated in the Results section with data visualizations.

RESULTS:

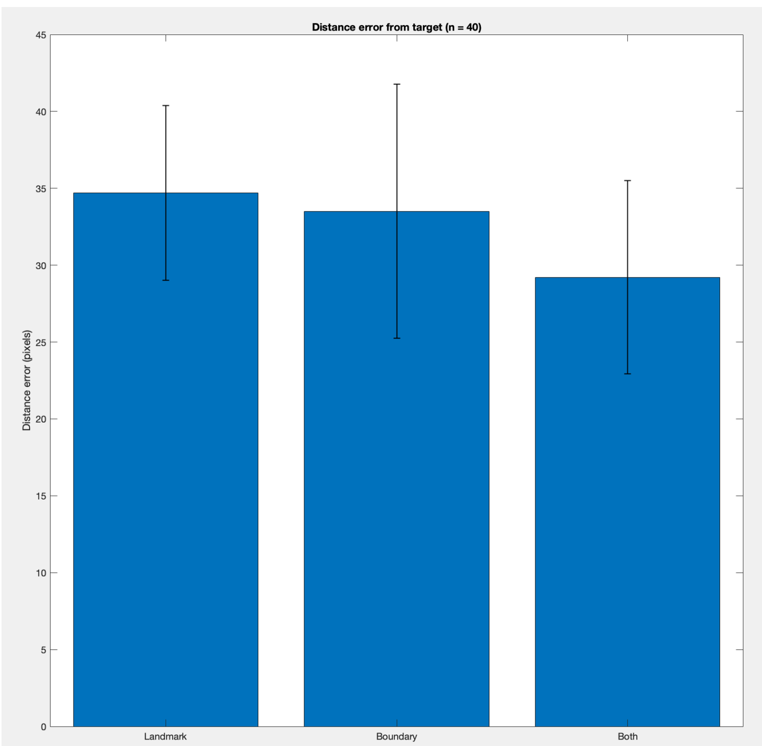

Figure 2. We began the analysis with a basic error calculation to determine how accurately participants attempted to place the Target across conditions. We took the Cartesian difference between the Target and the Attempt for each trial and separated the results into the three conditions. The Both condition had significantly less error than Landmark and Boundary only conditions.

Average Error by Condition:

The results of the experiment are summarized in Figure 2. First, we examined overall accuracy. Across all trial types, participants erred by an average of 32.5 pixels (*SD*=6.0 pixels). Then we compared accuracy across the three trial types ('landmark', 'boundary', 'both'). In the 'landmark' trials, participants erred by an average of 34.6950 pixels (*SD*=5.7 pixels); in the 'boundary' trials, participants erred by an average of 33.5070 pixels (*SD*=8.3 pixels); and in the 'both' trials, participants erred by an average of 29.2162 pixels (*SD*=6.3 pixels). Participants were more accurate in the 'both' trials than in the 'landmark' trials, t(39)=6.2, p<.001 (= p), *SD*=5.5844, as well the 'boundary' trials, t(39)=: 6.5939, *p*<.001, *SD*= 4.1155. The 'landmark' and 'boundary' trials did not differ from each other, t(39)=1.0987, *p*=0.2786, *SD*=6.8392. The increased accuracy in the 'both' trials indicates that participants benefited from being able to use both the landmark and the boundary to localize an object; the lack of a difference between the 'landmark' and the 'boundary' trials indicates that neither the landmark nor the boundary alone was more beneficial.

The results in Figure 2. suggest that the information available at the time of retrieval influences the magnitude of errors that participants make. However, our key question is whether the information available at the time of retrieval affects the *format* with which participants recall the relevant information, and how the format affects results. This means we have to not only analyze the magnitude of errors made but distinguish how participants erred between trials.. We take several approaches, first considering the raw data (*Figure 3*) then making a series of calculations for gravitation to specific visual elements within each condition (*Figures 4, 5, 6, 7*) as well as error correlation (see Yousif & Keil, 2021; for more information, see Methods).

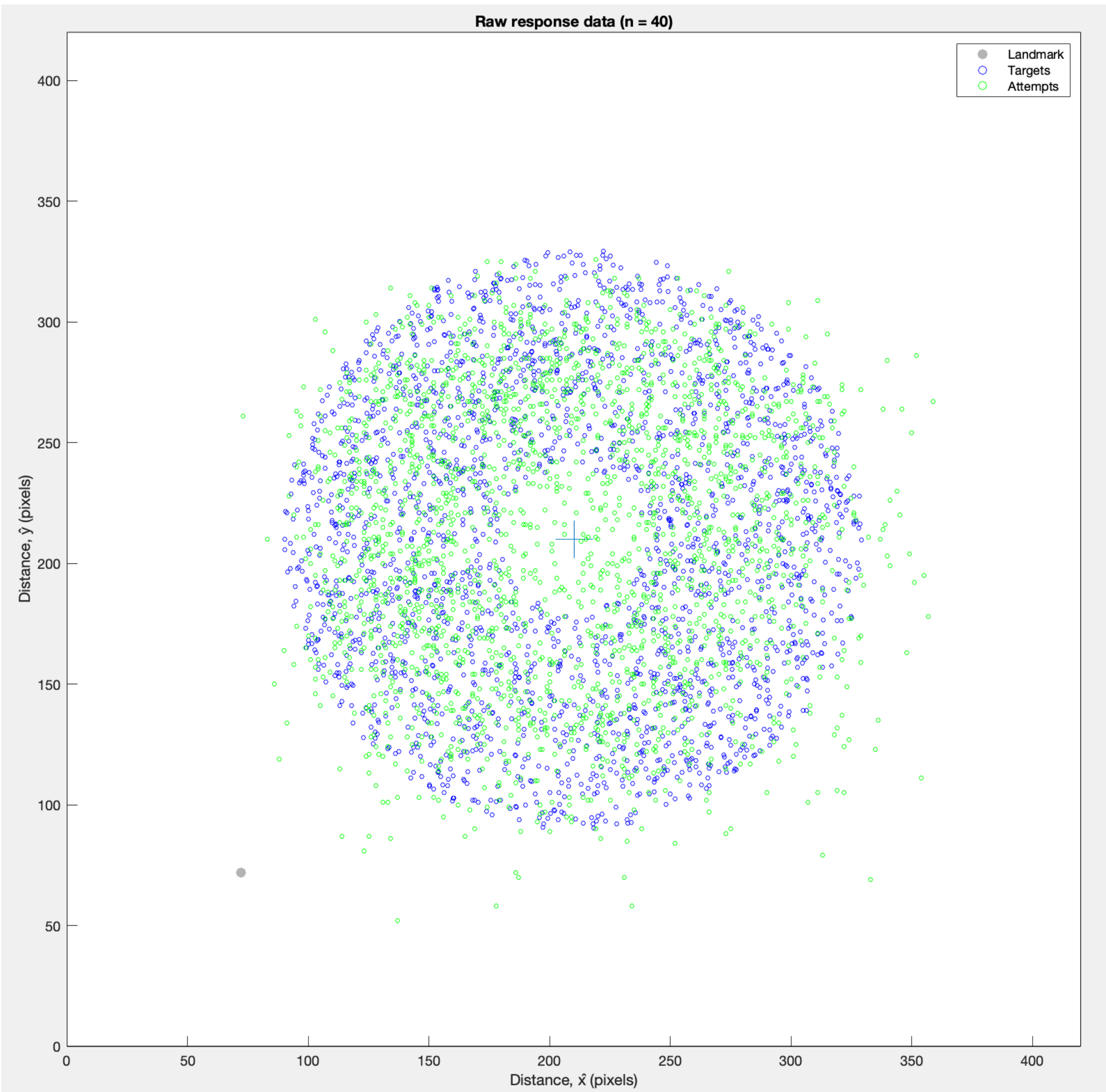

Figure 3. Raw data visualization. It was essential to first understand the structure of our raw data to begin meaningful manipulation. The figure above is a straightforward plot of the 420 pixel x 420 pixel box featured in each trial, within which we find the Landmark approximately 195 pixels southwest of the center of the grid. The blue dots represent the Targets and the green dots represent the Attempts of participants to replicate the location of the target. Overall we see the ‘donut’ shape of our randomly generated target points in the center of the square. We see

radial outliers predominantly to the south and east of the Targets. In the Discussion section, we speculate why this might be.

Landmark Gravity:

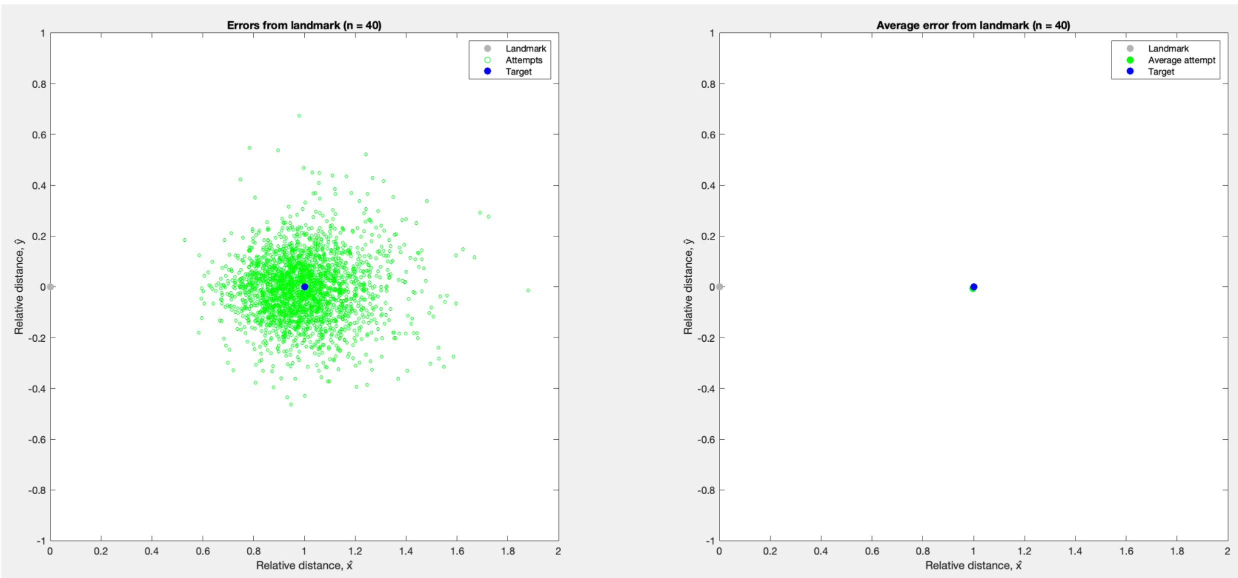

Figure 4. Normalized plots of Landmark gravitation. The left graph shows participant Errors from the landmark across all trials. These trials are averaged in the right graph. There is a statistically significant pull due west toward the Landmark at (0,0).

To compare the results of each trial, we needed to first standardize and transpose the raw data. The Targets, as visible in the raw visualization (Figure 3), were placed randomly in the space for each trial and so the raw responses could not be immediately averaged. To correct for this, we set the distance between the Landmark and Target to a standardized 1.000 unit and reduced the distance between the Landmark and Attempt proportionally. The angle created by the Landmark, Target, and Attempt needed not to be manipulated. Each trial was subsequently plotted with the Landmark at (0.000,0.000), the Target at (1.000,0.000), and the Attempt at its relative position, determined by the proportional distance and original angle relative to the Landmark and Target. The result of such manipulation permits relative comparison of Attempts around a singular Target. We can render two visualizations of this analysis, one with all Attempts charted on the unit graph, and another with a singular Attempt Average. Both visualizations allow us to see whether the Landmark exerts some gravitational pull on the Attempts by pulling them in its direction. The measure which we use to quantify the strength of this anchor is the distance between the Landmark and the Average Attempt, represented as a proportion of 1.000, where < 1.000 demonstrates an active gravitational pull. We found that the Landmark anchor produced a gravitational pull that reduced the average distance between the Landmark and Attempt to 99.69% of the distance between the Landmark and the Target. This pull, while notable in terms of its directionality, was not statistically significant, *t(39) = -0.9697, p = 0.3323, SD = 0.1569 (see Figure 4.)*.

Boundary Gravity:

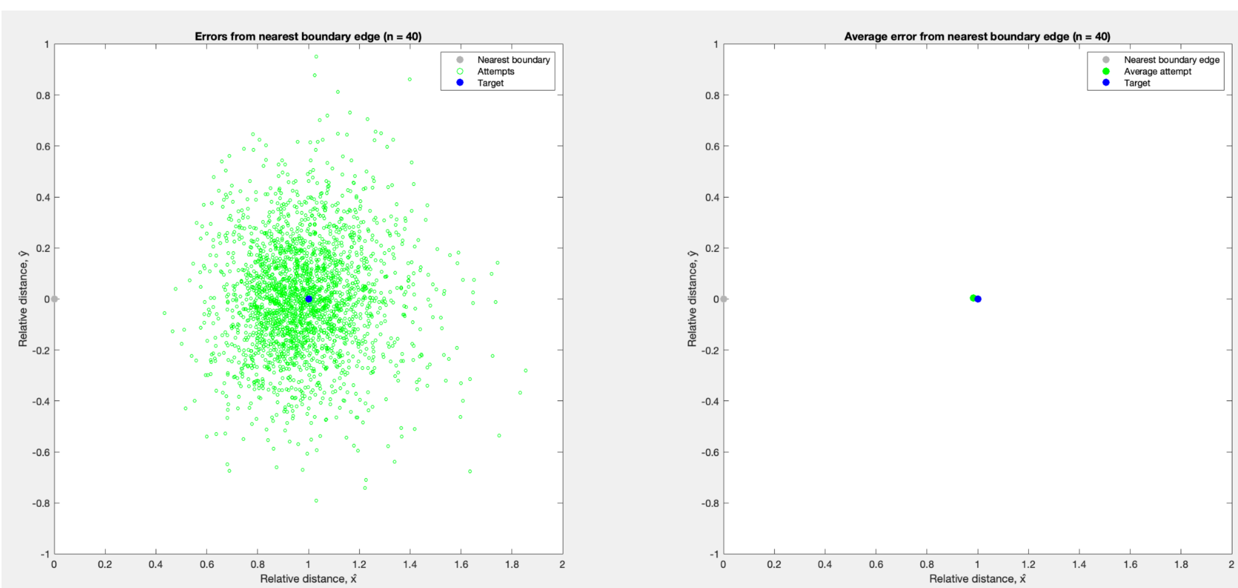

Figure 5. Normalized plots of Boundary Edge gravitation, i.e. gravity toward the point of the boundary nearest to the Target. The left graph shows participant Errors from the boundary across all trials. These trials are averaged in the right graph. There is a statistically significant pull toward the Boundary Edge at (0,0).

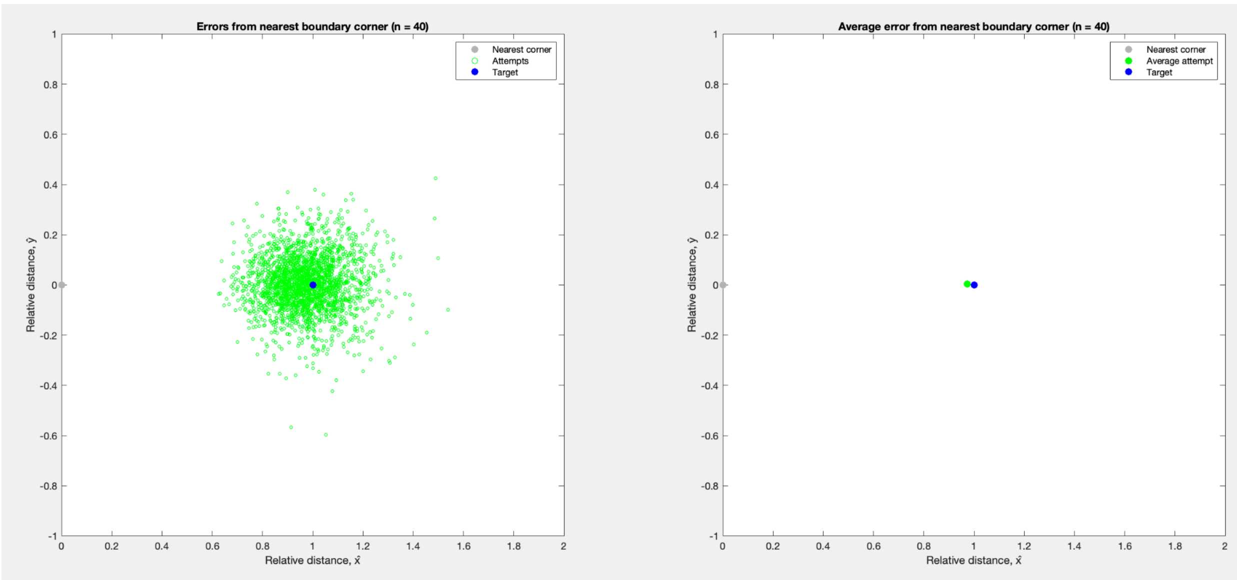

Figure 6. Normalized plots of Boundary Corner gravitation, i.e. gravity toward the corner of the boundary nearest to the Target. The left graph shows participant Errors from the boundary across all trials. These trials are averaged in the right graph. There is a statistically significant pull toward the Boundary Corner at (0,0), the strongest of our study.

The standardization and transposition procedures used to produce the graphical representation in the Landmark trials are repeated here for the Boundary trials. However, the position of this anchor is not as straightforward, as there is no *singular* point that would evidently appear to exert a gravitational force on the participants' Attempts. There might be some attraction toward the Boundary, but we needed to determine some particular point to test that attraction. We hypothesized two potential anchors that might factor into the mental representation of a Boundary: the *point* of the Boundary nearest to the Attempt ("Boundary Edge"), and the *corner* of the Boundary nearest to the Attempt ("Boundary Corner''). We found that the Boundary Edge anchor produced a gravitational pull that reduced the average distance between the Landmark and Attempt to 98.40% of the distance between the Landmark and the Target *(See Figure 5.).* This pull was statistically significant, $t(39)$= -4.1794, p < .001, SD = 0.1862. We then found that the Boundary Corner anchor produced an even stronger gravitational pull that reduced the average distance between the landmark and the Attempt to 97.36% of the distance between the Landmark and the target. This pull was also statistically significant, $t(39)$= -10.3535, p < .001, SD = 0.1235 *(See Figure 6).*

Landmark + Boundary Gravity (Both):

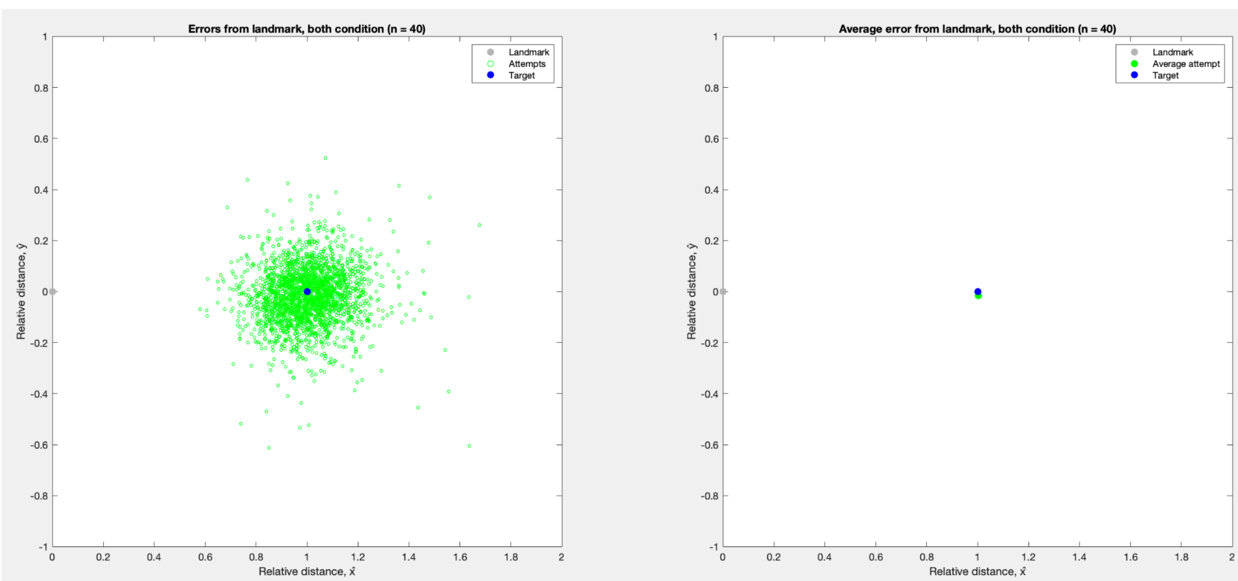

Figure 7. Normalized plots of Landmark gravitation in the Both condition, where a Boundary was also present. The left graph shows participant Errors from the landmark across all trials. These trials are averaged in the right graph. There is no significant pull away from the Landmark at (0,0).

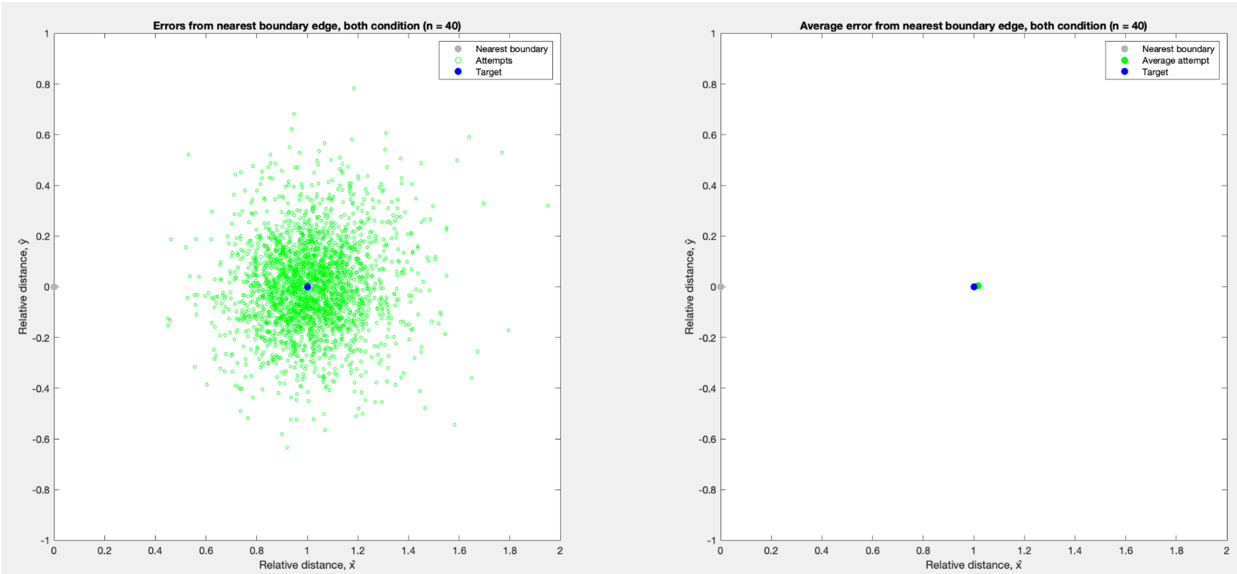

Figure 8. Normalized plots of Boundary Edge gravitation in the Both condition, where a Landmark was also present. The left graph shows participant Errors from the landmark across all trials. These trials are averaged in the right graph. There is a significant push away from the Boundary Edge at (0,0).

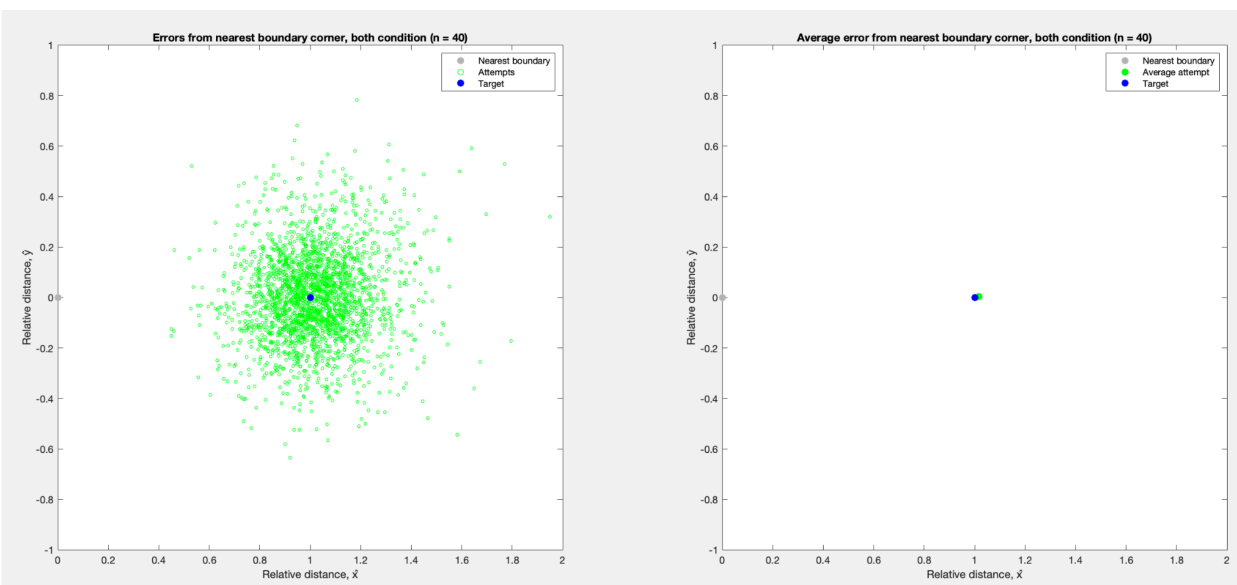

Figure 9. Normalized plots of Boundary Corner gravitation in the Both condition, where a Landmark was also present. The left graph shows participant Errors from the landmark across all trials. These trials are averaged in the right graph. There is no significant pull toward from the Boundary Corner at (0,0).

For the trials that included both the Landmark and the Boundary, we ran analyses of the Attempts' relative proximities to the Landmark, the nearest Boundary edge, and the nearest Boundary corner. We used the same standardization and transposition techniques as in the previous analyses. We found that the Landmark anchor did not produce a gravitational pull, as the average distance between the Landmark and the Attempt was 100.36% of the distance between the Landmark and the Target. This repulsion, however, was not statistically significant, $t(39)=1.3944$, $p=0.1633$, $SD=0.1242$ *(Figure 7).* We also found that the Boundary Edge anchor did not produce a gravitational pull either, but instead, a statistically significant gravitational push away from the anchor, resulting in a 101.76% increase in the distance between the Landmark and the Attempt relative to the distance between the Landmark and the Target, $t(39)=5.0496$, $p<.001$, $SD=0.1687$ *(Figure 8).* Interestingly, the Boundary Corner anchor produced a contradictory gravitational pull that slightly decreased the average distance between the Landmark and the Attempt to 99.80% of the distance between the Boundary Corner and the Target. This pull was not statistically significant, $t(39)=-0.8986$, $p=0.3690$, $SD=0.1077$, and was consequently outweighed by the repellent force of the Boundary Edge *(Figure 9.)*.

Overall, the technique of standardization and transposition allows us to see how participants made their errors across different conditions. We can apply this further to cartesian and polar formats by observing error correlations.

Error Correlations:

We analyzed error correlations for both Cartesian coordinates and polar coordinates for each of the three conditions. There are many different patterns of results we may expect. First, in the 'landmark' condition, we should expect that Cartesian errors are *correlated* and that polar errors are uncorrelated. Prior work using the 'error correlation' method has shown that in simple displays with only a landmark, participants spontaneously rely on polar coordinates. In the 'boundary' condition, we might expect the opposite: prior work has also shown that in spaces that more strongly imply Cartesian coordinates (e.g., squares, grid; Yousif & Keil, 2021), patterns of error correlations flip so that polar errors are correlated and Cartesian errors are not, potentially indicating the use of Cartesian coordinates. In the 'boundary' condition, it is not clear what we should expect. On any given trial, a participant may use the landmark, the boundary, or both. Thus, we have no clear predictions about the patterns of error correlations that we should expect. However, if we observed a correlation for Cartesian and not polar, that may indicate that people are relying on polar coordinates relative to the landmark; and if we observed the opposite, that may indicate that people are relying on Cartesian coordinates relative to the boundary.

We note that these predictions are speculative, as 'error correlations' are noisy, and it is not always clear, even in statistical simulation, what patterns we should expect. There are certain circumstances where, depending on how Cartesian coordinates are implemented, for instance, we might expect both Cartesian and polar errors to be uncorrelated, or only Cartesian errors to be uncorrelated.

The patterns of error correlations are summarized below:

Starting with the 'landmark' condition, Cartesian errors were correlated on average, $t(39)=5.3849$, $p<.001$, $SD= 0.155$, whereas Polar errors were not, $t(39)= -1.7667$ $p= 0.0851$, $SD=0.1259$.

In the 'boundary' condition, Cartesian errors were not correlated, $t(39)=0.6722$, $p=0.5054$, $SD=0.1116$, and the polar errors were also not correlated, $t(39)=1.1103$, $p=0.2737$, $SD=0.1160$.

Finally, in the 'both' condition, Cartesian errors were not correlated on average, $t(39)=3.2007$, $p=0.0027$, $SD=0.1515$, whereas polar errors were, $t(39)=-4.3859$, $p<.001$, $SD=0.1287$.

Overall, for the landmark condition, our hypothesis was correct that Cartesian errors are *correlated* and that polar errors are uncorrelated. These results could be interpreted as a spontaneous use of a polar coordinate system for solving the task in the landmark-only condition. For the boundary condition, our hypothesis that polar errors are correlated and Cartesian errors are not was not totally correct, since both were not correlated. However, from the results it at least seems to indicate that the polar system is not clearly being used, implying a change in format.

DISCUSSION:

Key Findings:

Consistent with recent studies, as well as a long tradition in cognitive science, we find that it is possible to recover information about the *format* in which people represent spatial environments from small errors that participants make in a simple localization task (Yousif 2021). The design of the task was based on the popularly accepted idea of spatial navigation relative to landmarks and boundaries (Doeller et al. 2008). In our study, we go one step further and suggest that one reason this might be the case is because landmarks and boundaries invoke different underlying spatial formats, which can be held in the mind simultaneously. Key results of our study support this:

1. Participants' accuracy when they had only the landmark or boundary was equal, but their accuracy increased significantly when they had both. This means that both forms of information (landmark and boundary) are contributing to the recollection of the dots' location, rather than a preference being given to one or the other. This implies that both are being encoded simultaneously.
2. In the trials that are only landmarks or boundaries, attempts show patterns of the gravitational pull of points within these graphic conditions, also referred to as anchors.
    a. Landmark - Although not significant, participants' attempts did lean closer to the landmark when only it was present, and further from it when only the

boundary was shown. Especially considering the raw data's outlying preference towards the bottom right corner, the placement of attempts in the direction of the landmark (which was in the bottom left corner) seems like an effect of the landmark, and the use of a polar coordinate system.

b. Boundary - The boundary edges and corner conditions both showed significance. If we assume that people click closer to the elements they are looking at, these results imply that a geometric format based on vertices between the boundaries were being deployed. This format could resemble a Cartesian coordinate system.

c. Both - We found that when the boundary was present, the landmark's gravitational effect was not observed. The Boundary Edge created a significant gravitational push away from it, while the Boundary Corner created a non-significant pull towards it. These conflicting results may nullify the possibility of the boundary being the dominant anchor for formatting. What is clear is that when both conditions are present, people's responses are no longer consistent with their results when only one is present. This implies a change in how format is being used.

3. Using an error correlation approach, we showed that in each case the patterns of errors were distinctive. When participants only had access to the landmark at the point of retrieval their patterns of errors were more consistent with the use of a polar coordinate system. When participants only had access to the boundary or both conditions their errors did not show a preference for a polar system.

In combination, these results indicate that formats are very sensitive and adaptable to changing conditions, yet the replicability in their errors indicates the presence of an underlying system that compulsively deploys spatial representations.

Dynamic Formatting:

Early discussions of mental representation argue for one standard format for mental representation, creating a long history of researchers favoring and debating the dominance of one format or another (Yousif 2022). The results of this study support a more dynamic view. It has been shown that people could hold two different kinds of formats in their mind at different times (Yousif 2021). However, the design of this study didn't allow the participant to anticipate which of the two formats would be necessary, requiring them to maintain both in mind at once. The element of time in this study, and the way overlapping representations are held in the same instance, can begin to give an idea of how the mind uses a variety of spatial formats at once, to represent different scenes and situations.

We experience various mental representations of the world daily, whether they be spatial, social, rooted in memories, the immediate environment, or imagined futures. Format is a state in which space in the mind is organized. Ultimately, there can be multiple types of mental representations of spatial structure, map-like, graph-like, and skeletal (Pier 21). Given tangible progress made, in this study and others, spatial representations of location should be considered a promising ‘case study’ for how the mind simultaneously renders and organizes various forms of information. Furthermore, it appears that space itself could be the *medium* in which many of these representations are held and recalled (Peer 21).

Future Directions

This study could be expanded by creating more variation between the graphical elements shown. For the landmark, a future study should observe if results, specifically the gravitation towards the landmark and the use of a polar format are still present if the location of the landmark (within the boundary) moves location (in this study the landmark stayed in the same location relative to the boundary). For variation in the boundary condition, a future study could observe if accuracy or the use of a Cartesian system is compromised if the shape of the boundary is converted to a non-grid-like shape, such as a circle or irregularly shaped boundary.

Looking at the raw data (*Figure 3*) we see radial outliers predominantly to the south and the east of the Targets. We could speculate that this may be due to the position of the landmark (which was fixed on the southwest side) and or the position of the mouse on the computer, which was on the right-hand side. A future study could consider flipping these conditions to see if there is a similar effect.

The influence of other spatial 'boundaries' in the environment, such as the screen or the buildings' floor plan, could also be considered. Would polar coordinate systems be more favored on a circular screen? Lastly, future studies could push the time element further.

What is the minimum amount of time possible for the average person to still hold two mental formats? Does a longer encoding time increase the number of mental formats one can hold?

Lastly, I want to consider that a study like this truly reveals itself in the analysis, and most importantly in the interpretation of the data. I would be curious to hear feedback about what certain results mean about other aspects of mental representation, or how participants were potentially thinking about solving the task. As part of my study I collected some debriefing notes from participants in the study, and would be curious to interpret those, and specifically associate results with participants' who indicated a unique form of solving the spatial task. This would be in response to the strong intuition I have that there are significant individual differences between different people. In working with averages our study did not take this into account.

Conclusion:

People have considered spatial formats in a variety of ways, but perhaps the most fundamental of all spatial representations is the representation of location. In this simple spatial study, we have shown that location can be represented in at least two ways, at the same time, implying that the mind could represent other broader forms of spatial representation in a similar, or even more complex way than previously expected. While mental representations are still far from understood, the field of spatial cognition seems like a promising place to continue laying tracks.

Acknowledgements:

I would like to thank my research mentor Sami Yousif for taking me under his wing this last year as we have conceived, created and executed this study together. I am constantly in awe of Sami's technical skills, seriousness as a mentor, as well as his emotional sensibility. He has taken the time to truly get to know me and how to work with me as a person, finding the middle ground of being instructional and collaborative. I feel happy for all the people that will get to work with him in the future.

I would also like to thank my research advisor Frank Keil for allowing me to run this study through the Yale Cognition and Development Lab. I am grateful for Frank's consistent support of this work and thoughtful feedback. His magical presence in the classroom is an inspiration for me, and many of his students.

Lastly, I would like to thank statistician and friend Jarett Malouf for his assistance and passion for the results of this study. Jarett's contributions as a computer and cognitive scientist allowed us to extract even more interesting conclusions and visualizations from our data.

WORKS CITED:

Ayzenberg, V., Yousif, S., & Lourenco, S. (2016). The medial axis as a robust model of object representation. *Journal of Vision*, *16*(12), 169-169.

Bays, P. M., Wu, E. Y., & Husain, M. (2011). Storage and binding of object features in visual working memory. *Neuropsychologia*, *49*(6), 1622-1631.

Bullens, J. (2009). *How children learn to deal with space: Developmental studies on spatial memory* (Doctoral dissertation, University Utrecht)

Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology: General, 122*(3), 371–396. https://doi.org/10.1037/0096-3445.122.3.371

Doeller, C. F., King, J. A., & Burgess, N. (2008). Parallel striatal and hippocampal systems for landmarks and boundaries in spatial memory. *Proceedings of the National Academy of Sciences*, *105*(15), 5915-5920.

Firestone, C., & Scholl, B. J. (2014). “Please tap the shape, anywhere you like” shape skeletons in human vision revealed by an exceedingly simple measure. *Psychological science*, *25*(2), 377-386.

Hartley, T., Lever, C., Burgess, N., & O'Keefe, J. (2014). Space in the brain: how the hippocampal formation supports spatial cognition. *Philosophical Transactions of the Royal Society B: Biological Sciences*, *369*(1635), 20120510.

Kelly, J. W., McNamara, T. P., Bodenheimer, B., Carr, T. H., & Rieser, J. J. (2008). The shape of human navigation: How environmental geometry is used in maintenance of spatial orientation. *Cognition*, *109*(2), 281-286.

Kosslyn, S. M., Thompson, W. L., & Ganis, G. (2006). *The case for mental imagery*. Oxford University Press.

Kuipers, B. (1978). Modeling spatial knowledge. Cognitive Science, 2, 129-153.

Kuipers, B. (1982). The" map in the head" metaphor. Environment and Behavior, 14, 202-220.

Krupic, J., Bauza, M., Burton, S., & O'Keefe, J. (2016). Framing the grid: effect of boundaries on grid cells and navigation. *The Journal of physiology*, *594*(22), 6489-6499.

O'keefe, J., & Nadel, L. (1979). Précis of O'Keefe & Nadel's The hippocampus as a cognitive map. *Behavioral and Brain Sciences*, *2*(4), 487-494.

Parkinson, C., & Wheatley, T. (2013). Old cortex, new contexts: Re-purposing spatial perception for social cognition. *Frontiers in human neuroscience*, *7*, 645.

Peer, M., Brunec, I. K., Newcombe, N. S., & Epstein, R. A. (2020). Structuring Knowledge with Cognitive Maps and Cognitive Graphs. Trends in Cognitive Sciences. 25. 37-54.

Tolman, E. C. (1948). Cognitive maps in rats and men. *Psychological Review, 55*(4), 189–208. https://doi.org/10.1037/h0061626

Tversky, B. (1993, September). Cognitive maps, cognitive collages, and spatial mental models. In European Conference on Spatial Information Theory (pp. 14-24). Springer, Berlin, Heidelberg.

Yousif, S. R., & Keil, F. C. (2021). The shape of space: Evidence for spontaneous but flexible use of polar coordinates in visuospatial representations. *Psychological Science*, *32*(4), 573-586.

Yousif, S. R. (2022). Redundancy and reducibility in the formats of spatial representations. *Perspectives on Psychological Science*, *17*(6), 1778-1793.

Yousif, S. R., & Keil, F. (2021). 'Decoding' the locus of spatial representation from simple localization errors. In *Proceedings of the Annual Meeting of the Cognitive Science Society* (Vol. 43, No. 43).

Warren, W. H., Rothman, D. B., Schnapp, B. H., & Ericson, J. D. (2017). Wormholes in virtual space:From cognitive maps to cognitive graphs. Cognition, 166, 152-163.

Zeman, A., Dewar, M., & Della Sala, S. (2015). Lives without imagery–Congenital aphantasia. *Cortex*, *73*, 378-380.