EN.530.335 Heat Transfer Laboratory
Design Project Final Report
Experiments Conducted on April 18, 2023
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Cost Report

The total cost of the heat exchanger came out to be \$94.11. The total budget we were allowed to spend was \$150, and we spent \$148.83. There were no manufacturing costs because all manufacturing was done with hand tools.

Table 1: All of the purchases made throughout the semester and the costs contributing to the final heat exchanger. All cost values are in units of USD.

Part	Total Cost	Unit Cost	Quantity Used in Final Heat Exchanger	Contribution to Heat Exchanger Cost
Otdorpatio Project Box	21.99	21.99	1	21.99
6ft Copper Tubing ¼ Tube Size	56.64	4.72/ft	4.875ft	23.01
Brass Push-to-Connect Fitting Adapter	13.30	6.65	2	13.30
EPDM Gasket	4.16	0.52	4	2.08
Polycarbonate Sheet	14.63	0.102/in^2	133.92in^2	13.66
Stainless Steel Washer	8.83	8.83	0	0
Zinc- Plated Steel Hex Nut	3.11	0.031	6	0.19
L Bracket Brace	8.99	0.45	6	2.70
Hex Lock Nut ½ Inch NPT Female	17.18	4.295	4	17.18

Modeling

For our computational model, we aimed to calculate the outlet temperatures, pressure drops, and heat transfer rates for both the hot and cold water. Our heat exchanger was inspired by a one-shell-pass shell and tube design, and so our matlab model was built with known equations that corresponded to that model type.

To simplify our program we took several assumptions to be true. We assumed that no heat would transfer to and from the heat exchanger surroundings, and that the water properties would not change from their initial values. We also assumed that the conduction through the thin pipe walls was negligible, and that the surface through which heat transferred was the outer surface of the copper pipe.

The hot water heat transfer coefficient h_{hot} ($\frac{W}{m^2 \cdot K}$) was based on the following equations for internal flow in a circular pipe,

$$h_{hot} = \frac{Nu_{hot} \times k_{hot}}{D_{in}} \tag{1}$$

$$Nu_{hot} = \frac{(f/8)(Re_{hot} - 1000)Pr_{hot}}{1 + 1.27(Pr_{hot}^{2/3} - 1)\sqrt{f/8}} \tag{2}$$

$$f = (0.790 ln(Re_{hot} - 1.64))^{-2} \tag{3}$$

$$u_{hot} = \frac{4Q}{\pi D_{in}^2} \tag{4}$$

where Nu_{hot} is the hot water Nusselt number, f is the friction coefficient for the copper pipe, Re_{hot} is the Reynold's number for the hot water, Pr_{hot} is the hot water Prandtl number, u_{hot} is the average velocity of the hot water ($\frac{m}{s}$), Q is the volumetric flow rate of both water types ($\frac{m^3}{s}$), and D_{in} is the inner diameter of the copper pipe (m). Equations for the Prandtl number and Reynold's number are the same for both water temperatures. The cold water heat transfer coefficient h_{cold} ($\frac{W}{m^2 \cdot K}$) was based on the following equations for external flow in a circular shell with baffles,

$$h_{cold} = \frac{Nu_{cold} \times k_{cold}}{D_h} \tag{5}$$

$$Nu_{cold} = 0.683 Re_{cold}^{0.466} Pr_{cold}^{1/3} \quad (6)$$

$$u_{cold} = \frac{Q}{D_{hd}} \quad (7)$$

where Nu_{cold} is the cold water Nusselt number, Re_{cold} is the Reynold's number for the cold water, Pr_{cold} is the cold water Prandtl number, u_{cold} is the average velocity of the cold water $\left(\frac{m}{s}\right)$, and D_{hd} is the characteristic length of the shell (m).

Using the values from Equations (1) and (5), the overall heat transfer coefficient U $\left(\frac{W}{m^2 \cdot K}\right)$, the number of heat transfer units NTU, and the heat transfer effectiveness ε were obtained as follows,

$$U = \left(\frac{1}{h_{cold}} + \frac{1}{h_{hot}}\right)^{-1} \quad (8)$$

$$NTU = \frac{UA}{c_{min}} \quad (9)$$

$$\varepsilon = 2\left\{1 + C_r + \left(1 + C_r^2\right)^{1/2} \times \frac{1+exp[-(NTU)\left(1+C_r^2\right)^{1/2}]}{1-exp[-(NTU\left(1+C_r^2\right)^{1/2}]}\}\right\}^{-1} \quad (10)$$

where A is the outer pipe surface area $\left(m^2\right)$, c_{min} is the smaller specific heat capacity between that of the cold water and that of the hot water $\left(\frac{J}{kg \cdot K}\right)$, and C_r is the ratio of c_{min} over the larger specific heat capacity [1].

With the variables obtained thus far, it is now possible to calculate the rate of heat transfer out of the hot water q_{hot} (W), the rate of heat transfer into the cold water q_{cold} (W), and

[1] Equation 10 is from Bergman T.L. and Lavine A.S., Fundamentals of Heat and Mass Transfer, published by Wiley, eighth edition.

the outlet temperatures $T_{hot,\,out}$ and $T_{cold,\,out}$ (K). We use Equations (11), (12), and (13) as follows[2].

$$q_{hot} = \varepsilon q_{max} = \varepsilon c_{min}\left(T_{hot,\,in} - T_{cold,\,in}\right) \tag{11}$$

$$q_{cold} = \eta q_{hot} \tag{12}$$

$$T_{i,\,out} = T_{i,\,in} - \frac{q_i}{C_i} \tag{13}$$

where $T_{i,\,out}$ is the outlet temperature (K), $T_{i,\,in}$ is the inlet temperature (K), C_i is the specific heat capacity $\left(\frac{J}{kg\cdot K}\right)$, q_i is the heat transfer rate for the respective water type (W), and η is the heat exchanger efficiency. It is important to note that η is equal to 1 since we assume that any and all heat transfer will occur only between the two types of water through a negligibly thin pipe wall.

Finally, our model calculates the pressure drop ΔP (Pa) for both the hot and cold water as described by Equation (14) below,

$$\Delta P = \Sigma_i \frac{1}{2} f \rho_i u_i^2 \frac{L_i}{D_i} \tag{14}$$

where ρ_i is the density of the water (Pa), u_i is the average velocity of the water ($\frac{m}{s}$), L_i is the straight length distance that the water travels (m), and D_i is the characteristic length of the flow path's cross sectional area. The summation symbol represents the summation of all types of pipe orientations that a particular water type will flow through, such as 90° turns and 45° turns.

The model's predicted value for q_{hot} is 240.6 W.

Results

[2] Note that the i subscripts in Equations 11, 12, and 13 represent the values for the hot or cold water. These equations will work for both water types.

Table 2: Averaged measured values of the raw temperature, flow rate, and pressure drop.

	Hot Water	Cold Water
Temperature Inlet (°C)	30.0	6.10
Temperature Outlet (°C)	25.8	11.4
Volumetric Flow Rate (l/min)	2.0	2.4
Pressure Drop (kPa)	10.03	1.71

Table 3: Measured values from custom heat exchanger and calculated cost.

Mass of Empty Heat Exchanger(kg)	Volume of Smallest Bounding Box(m3)	Cost of Heat Exchanger
1.56	0.0104	94.11

Table 4: Calculated rates of heat removed and transferred and thermal efficiency

qh→ (W)	*q*→*c* (W)	η
568	742	1.31

Table 5: Calculated values from raw data for determining heat exchanger performance

U (W/m^2K)	ε	NTU
671.91	0.173	0.216

Table 6: Calculated figure of merits for both custom heat exchanger and the Armfield tubular heat exchanger

	F1 (W)	F2 (W/m^3)	F3 (W/kg)	F4 (W/kPa)	F5 (W/$)
Custom Heat Exchanger	568	54900	364	56.7	6.04
Armfield	497	17400	648	55.16	N/A

Discussion

The performance of our actual heat exchanger was much better than the predicted performance. The predicted heat transfer rate from the hot water was 57.7% lower than the actual rate. This is likely because the modeling equations we used in our code were not completely accurate for our design. Our heat exchanger was not a perfect one-shell-pass shell and tube heat exchanger. The traditional design has the hot and cold water flowing perpendicular to one another, while our design has the two flow paths continuously antiparallel. To improve the matlab code, it would be best to find a middle ground between the straight pipe counterflow modeling equations and the traditional shell and tube design equations.

Our heat exchanger outdid the Armfield Tubular heat exchanger in nearly all figures of merit, with the exception of heat transfer out of the hot water over mass. Our heat exchanger had a 14.3% higher heat transfer rate, a 216% higher heat rate to volume ratio, and a 2.8% higher heat rate to pressure drop ratio. Unfortunately, we did make our heat exchanger too heavy, as the Armfield exchanger had a 78.0% higher heat rate to mass ratio. While our exchanger performed generally better than the Armfield in terms of figures of merit, it is important to note that it did not function significantly better as a heat exchanger.

The actual efficiency was not 1, but rather 1.31. This is likely due to the heat transfer between the surroundings and the exchanger. Because our heat exchanger was running with the cold water in the shell, the cold water was exposed to the room temperature air surrounding the exchanger. Since our heat exchanger was not insulated, this allowed the cold water to absorb heat from the surroundings through the shell box, increasing the q_{cold} value far above what was predicted. Because the efficiency is calculated as the cold water heat transfer rate over hot water heat transfer rate, the efficiency was increased above 1.

Our design was limited by the length of pipe purchased and the cold water cross sectional area. Because of ordering delays, our team was cautious when it came to using our pipe. We did not want to run out and suddenly have to purchase more, so we did not plan to use all that we purchased. It is also riskier to create more complicated and harsh bends in the pipe. While this would have led to more turbulent flow and allow us to fit more pipe into the box, it also could have led to breakage that we were not prepared to fix. We focused on using our budget to buy materials and items that we did not have to severely alter. Therefore, we did not have a manufacturing part of our budget that we could rely on had we needed to solder broken pipes together. Our design was also limited in the way that it did not include more than one pathway for the hot water to travel. To increase our hot water heat transfer rate, it would have been beneficial to include more pipe paths and fill the shell cross sectional area with more pipe.

One of the changes made between the prototype design and final design was that the length of tubing was increased. This was done by adding bends in the straight sections of the pipe. While this did significantly increase the pipe length by about 60%, it may have been more effective to include multiple lines of pipe instead. This would have increased our pipe length by several times, which would have increased the heat transfer even if there was less cold water surrounding the pipes per unit cross sectional area. The only difference in cost would be due to the extra pipe, and since the cost of parts and manufacturing of the current heat exchanger was well below the $150 budget, the new design could be implemented without exceeding the $150 cost limit.

It also would have helped if we had had a non-manual pipe bender or if we had used 90° pipe fittings to connect the lines of straight pipes. The manual pipe bender was a hassle to use, sometimes broke our pipe, and limited the 180° bend radius that we could achieve. If there was a bender that gave a smaller bend radius or if we had used the aforementioned fittings, more piping could have fit within the box which would have improved the heat exchanger performance.

We learned to not overspend and to put more thought into the manufacturing aspect of our design. As previously stated, we opted for a less manufacturing heavy design, which ultimately led to us having to suddenly figure out how to manufacture our exchanger parts, rather than fully thinking through our building process. It also would have helped if we did not buy so much for our first prototype. We should have gathered small spare parts and built more iterations of our design so that we could slowly improve it over time, rather than rushing to fix problems as they arose. This also forced us to have a more “single fastest solution” approach to problem solving, rather than allowing us to brainstorm better and more creative ways to get past obstacles in our design. We suggest that there be a hard deadline for ordering materials, that the order form be available earlier, or that it is pushed on the students more to order most of their parts before the spring break.